Achieving Optimal Disturbance Rejection by Means of the Magnitude Optimum Method

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Abstract: The magnitude optimum method provides a non-oscillatory closed-loop response for a large class of process models. However, by applying this technique to low-order processes, the process poles could be cancelled by the controller zeros. This may lead to poor attenuation of load disturbances if the cancelled poles are excited by disturbances, and if they are slow compared to the dominant closed-loop poles. This paper shows that the described deficiency can be suppressed by slightly changing the optimisation criterion of the magnitude optimum method. This novel idea is supported by several simulations and one real-time experiment on a laboratory set-up.

Keywords: Disturbance Rejection, PID Control, Controller Tuning, Magnitude Optimum, Multiple Integration, Optimal Control.

1. Introduction

The Ziegler-Nichols tuning rules [10] were the very first tuning rules for PID controllers, and it is perhaps surprising that they are still widely used today. Their popularity lies in their simplicity and efficiency. This is why so many different tuning rules based on the same tuning procedures have subsequently been developed [3].

Following the work of Ziegler and Nichols, a variety of PID tuning methods have been developed. In general, these methods can be divided into two main groups: *direct* and *indirect* tuning methods [1, 3].

The direct tuning methods do not require a process model, while the indirect methods calculate controller parameters from an identified model of the process.

The direct methods are divided into rule-based methods and iterative search procedures. Controller tuning is usually performed in a closed-loop system. However, the tuning procedure is relatively long and requires an initially tuned controller and an a-priori defined "desired" closed-loop response.

The indirect tuning methods are usually based on the process model obtained from a transient (step) response or frequency response experiment. Therefore, the quality of the calculated controller parameters depends on the quality of the identified process model.

One of the indirect tuning methods is the magnitude optimum (hereafter "MO") method [4, 5], which results in a relatively fast and non-oscillatory system closed-loop response. However, the MO method is originally used for achieving superior reference tracking performance. On the other hand, by using the MO method, the process poles could be cancelled by the controller zeros. This may lead to poor attenuation of load disturbances if the cancelled poles are excited by disturbances and if they are slow compared to the dominant closed-loop poles [2]. Poorer disturbance rejection performance can be observed when controlling low-order processes. This is one of the most serious drawbacks of the MO method since, in process control, good disturbance rejection performance is usually more important than superior reference tracking performance.

However, as will be shown in this paper, "optimal" disturbance rejection properties can be achieved as well by slightly modifying the MO method.

2. Description of the MO Method

One possible objective when designing a control system is that the system's output instantaneously follows the reference. In other words, the closed-loop system should have an infinite bandwidth and zero phase shift. However, in practice this is not possible since every system contains some time delay and/or dynamics and controller gain is limited due to the physical limitations.

Therefore, the system's dynamics cannot be ignored, and a new design objective is needed. One possible design objective is to maintain the closed-loop magnitude response curve as flat and as close to unity for as large bandwidth as possible for a given plant and controller structure [4, 6]. Therefore, the idea is to find a controller that results in magnitude response flat and close to unity for as large a bandwidth as possible (see Fig. 1). It results in a fast and non-oscillatory closed-loop response for a large class of processes.

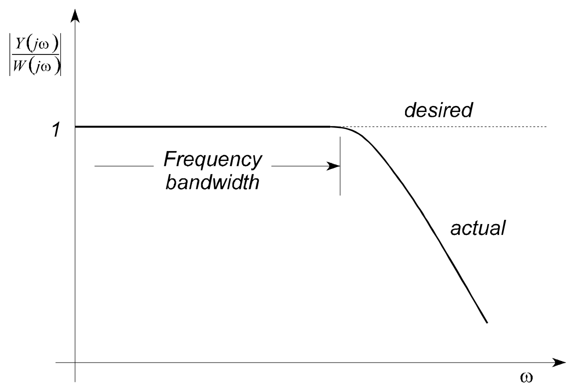

Fig. 1. Magnitude optimum (MO) criterion.

This technique is called magnitude optimum (MO) [6], modulus optimum [2], or Betragsoptimum [2, 5], and results in a fast and non-oscillatory closed-loop time response for a large class of process models.

If $G_{CL}(s)$ is the closed-loop transfer function from the set-point to the process output, the controller is determined in such a way that

$$G_{CL}(0)=1$$
$$\left.\frac{d^r |G_{CL}(i\omega)|}{d\omega^r}\right|_{\omega=0}=0 \tag{1}$$

for as many r as possible [2].

Let us assume that the actual process is described by the following transfer function:

$$G_P(s)=K_{PR}\frac{1+b_1 s+b_2 s^2+\cdots+b_m s^m}{1+a_1 s+a_2 s^2+\cdots+a_n s^n}\ , \tag{2}$$

where K_{PR} denotes the process steady-state gain, and a_1 to a_n and b_1 to b_m are the corresponding parameters ($m \leq n$) of the process transfer function, where n can be an arbitrary positive integer value.

The controller structure is chosen to be of the PI type (see Fig. 2), described by the following transfer function:

$$G_C(s)=\frac{U(s)}{E(s)}=K\left(1+\frac{1}{sT_i}\right). \tag{3}$$

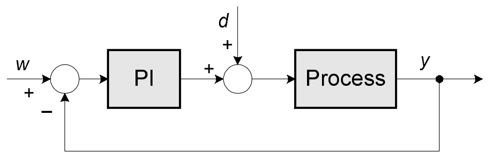

Fig. 2. The PI controller in the closed-loop with the process.

Then the PI controller parameters can be expressed by the unknown process parameters [7, 8, 9]:

$$K=\frac{\left[a_1^3-a_1^2 b_1+a_1 b_2-2a_1 a_2+a_2 b_1+a_3-b_3\right]}{2K_{PR}\left[-a_1^2 b_1+a_1 a_2+a_1 b_1^2-a_3-b_1 b_2+b_3\right]} \tag{4}$$

$$T_i=\frac{\left[a_1^3-a_1^2 b_1+a_1 b_2-2a_1 a_2+a_2 b_1+a_3-b_3\right]}{\left[a_1^2-a_1 b_1-a_2+b_2\right]} \tag{5}$$

The given tuning procedure will now be illustrated in two examples.

Case 1

The process is assumed to have the transfer function:

$$G_P=\frac{1}{(1+s)^5}\ . \tag{6}$$

The PI controller parameters, resulting from equations (4) and (5), are as follows:

$$K = 0.437,\ T_i = 2.33s \ . \qquad (7)$$

Fig. 3. shows the closed-loop responses on the reference change (w=1 at t=0s), and on the load-disturbance (d=1 at t=40s) under PI controller (7). Note that the closed-loop response features fast tracking and quite good disturbance rejection, which should be addressed to the MO criterion.

Case 2

The process is assumed to have the transfer function:

$$G_P = \frac{1}{(1+s)(1+0.1s)} \ . \qquad (8)$$

The PI controller parameters, resulting from equations (4) and (5), are as follows:

$$K = 5.05,\ T_i = 1.001s \ . \qquad (9)$$

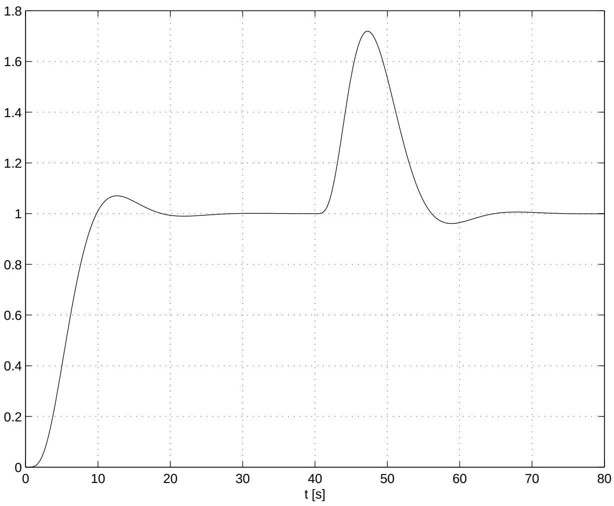

Fig. 3. The closed-loop response to the step change in reference signal and disturbance signal; the PI controller applied is tuned by the MO method.

Fig. 4. shows the closed-loop responses on the reference change (w=1 at t=0s), and on the load-disturbance (d=1 at t=5s) under PI controller (9). Note that the closed-loop response again features good tracking, but disturbance rejection is now quite sluggish. The reason is that the process is of a low order so the controller zero practically cancelled the slowest process pole.

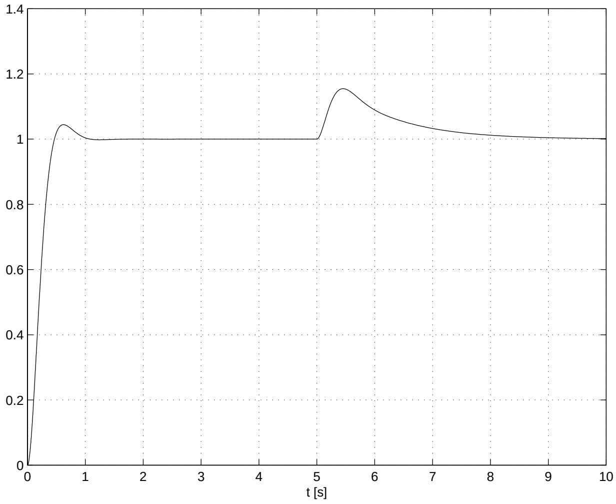

Fig. 4. The closed-loop response to the step change in reference signal and disturbance signal; the controller applied is tuned by the MO method.

3. Improving Disturbance Rejection by Means of the MO Method

In the previous section, it was shown that the MO method provides quite fast and non-oscillatory closed-loop response on reference change irrespective of the process order. On the other hand, disturbance rejection is degraded when dealing with low-order processes, since slow process poles are almost entirely cancelled by controller zero.

This is reasonable since the MO method aims at achieving good reference tracking, so it optimises the transfer function $G_{CL}(s)=Y(s)/W(s)$ and not the transfer function $G_{CLD}(s)=Y(s)/D(s)$. Let us now express $G_{CL}(s)$ in terms of $G_{CLD}(s)$:

$$\begin{aligned} G_{CL}(s) &= G_{CLD}(s)G_C(s) = G_{CLD}(s)\frac{K}{sT_i}(1+sT_i) = \\ &= G_{CLO}(s)(1+sT_i) \end{aligned} \ , \qquad (10)$$

where

$$G_{CLO}(s) = \frac{Y(s)}{D(s)}\frac{K}{sT_i} \ . \qquad (11)$$

From expression (10) it is clear that the controller's zero ($1+sT_i$) plays an important role within $G_{CL}(s)$, which is actually "optimised" by the original MO method (see Fig. 1). On the other hand, the controller's zero can significantly degrade disturbance rejection performance. A new strategy proposed herein is to optimise the transfer function $G_{CLO}(s)$ (11) instead of $G_{CL}(s)$ in expression (1).

The resulting PI controller parameters, when replacing $G_{CL}(s)$ with $G_{CLO}(s)$ in expression (1), can be expressed by the unknown process parameters:

$$K = \frac{-\beta + \mathrm{sgn}(\beta)\sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad ;if\ \alpha \neq 0 \qquad (12)$$

or

$$K = -\frac{\gamma}{\beta} \quad ;if\ \alpha = 0 \qquad (13)$$

and

$$T_i = \frac{2KK_{PR}(a_1 - b_1)}{1 + 2KK_{PR} + K^2 K_{PR}^{\ 2}}, \qquad (14)$$

where

$$\alpha = K_{PR}^{\ 3}\left(a_3 - b_3 - a_1 b_2 - a_2 b_1 + a_1 b_1^{\ 2} - b_1^{\ 3} + 2b_1 b_2\right) \quad (15)$$

$$\beta = 2K_{PR}^{\ 2}\left(a_3 - b_3 - a_1 a_2 + a_1^{\ 2} b_1 + b_1 b_2 - a_1 b_1^{\ 2}\right) \quad (16)$$

$$\gamma = K_{PR}\left(a_3 - b_3 + a_1 b_2 - 2a_1 a_2 + a_1^{\ 3} - a_1^{\ 2} b_1 + a_2 b_1\right). \quad (17)$$

4. Examples

Three simulated examples and one real-time experiment on a laboratory set-up were performed to depict the above results in more detail.

Case 3

The process is assumed to have the same transfer function as in Case 1. The PI controller parameters resulting from equations (12) and (14) are as follows:

$$K = 0.465,\ T_i = 2.17s\ . \qquad (18)$$

The calculated parameters are almost the same as those given in expression (7). It is therefore expected that disturbance rejection performance will remain almost the same. This assumption is confirmed by the closed-loop responses given in Fig. 5. Disturbance rejection properties remained quite good (solid line).

Case 4

The process is assumed to have the same transfer function as in Case 3. The PI controller parameters resulting from equations (13) and (14) are as follows:

$$K = 5.05,\ T_i = 0.304s\ . \qquad (19)$$

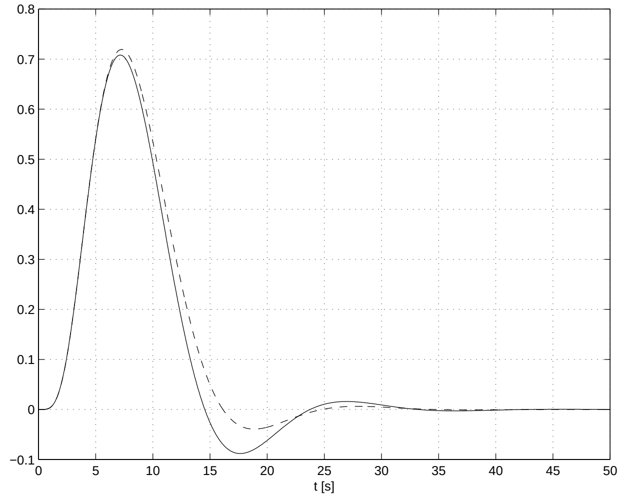

Fig. 5. The closed-loop response of the process (6) to disturbance signal (d=1 at t=0); __ PI controller (18), -- PI controller (7).

The calculated proportional gain is the same as that given in expression (9). On the other hand, the parameter of the integral term is now quite lower. The closed-loop responses on disturbance occurrence (d=1 at t=0s) are given in Fig. 6. Disturbance rejection properties are now quite improved (solid line) in comparison to the original MO method (dashed line).

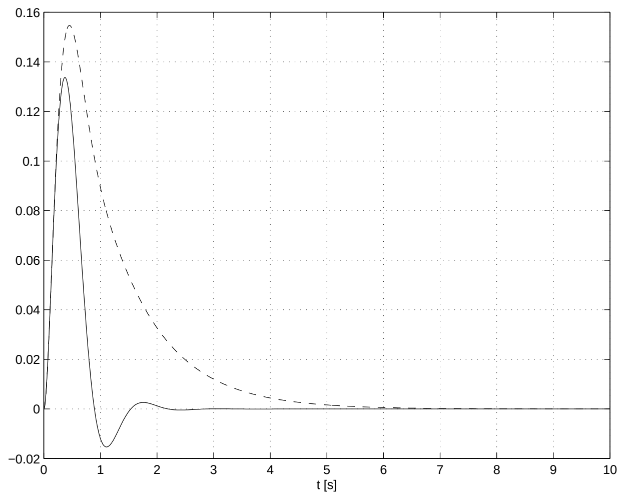

Fig. 6. The closed-loop response of the process (8) to disturbance signal (d=1 at t=0); __ PI controller (19), -- PI controller (9).

Case 5

The last experiment was conducted on the three-tank system shown in Fig. 7. The water comes from reservoir R_0 (see schematic diagram in Fig. 8) through pump P_1 to water column R_1. Valves V_1 and V_3 are closed. The process input is the voltage on the pump P_1, and the process output is the water level in water column R_1 (h_1).

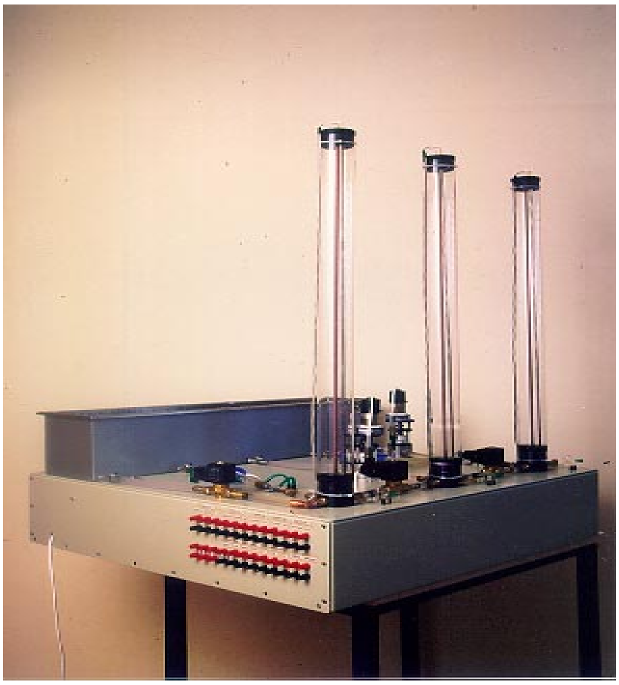

Fig. 7. Three-tank system.

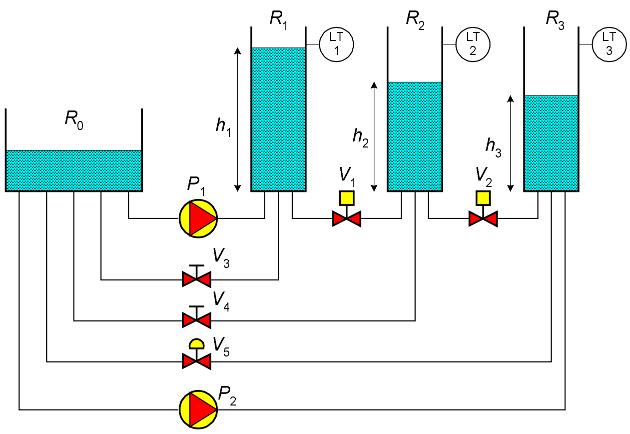

Fig. 8. Schematic diagram of the laboratory set-up.

Fig. 9. shows the open-loop process step response from which the following process parameters are estimated[i]: K_{PR}=2.151, a_1=28.729, a_2=164.34, a_3=0, b_1=b_2=b_3=0. The corresponding PI controller parameters are obtained from expressions (4) and (5), and are:

$$K = 0.702,\ T_i = 21.6s\ . \qquad (20)$$

The closed-loop response is shown in Fig. 10. The reference following seems to be very good, while disturbance rejection appears to be quite sluggish. In order to improve disturbance rejection performance, the PI controller parameters are re-calculated, using expressions (13) and (14):

$$K = 0.702,\ T_i = 13.8s\ . \qquad (21)$$

Fig. 11 shows improved disturbance rejection performance (solid-line) while using re-calculated PI controller parameters (21) in comparison to the performance achieved using the controller with the originally calculated parameters (dashed-line) (20).

5. Conclusions

The purpose of this paper was to present a modification to the original MO method in order to achieve “optimal” disturbance rejection performance. The given modification is simple and straightforward for implementation in practice. Simulation experiments on two types of process models have shown that the proposed approach results in very good disturbance rejection properties. The modified method was also tested in real-time on a laboratory set-up. It was shown that the proposed approach also works well in practice.

On the other hand, the original MO method, as well as the proposed modification, does not guarantee stable closed-loop responses [7, 9]. Unstable responses can be obtained when dealing with processes with oscillating poles or “strong” zeros [7]. Fortunately, such processes are rare in practice.

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[i] Estimation of the process parameters is made by means of the multiple integration method [7, 8, 9].

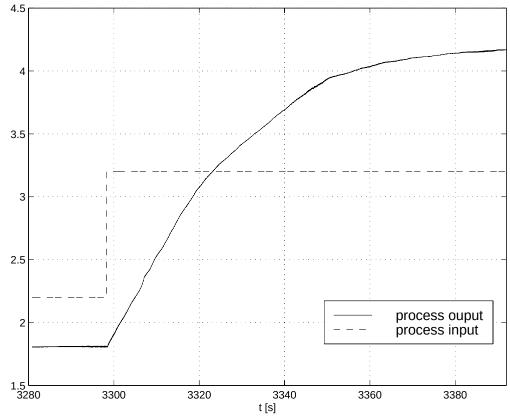

Fig. 9. The process open-loop response on input step-change.

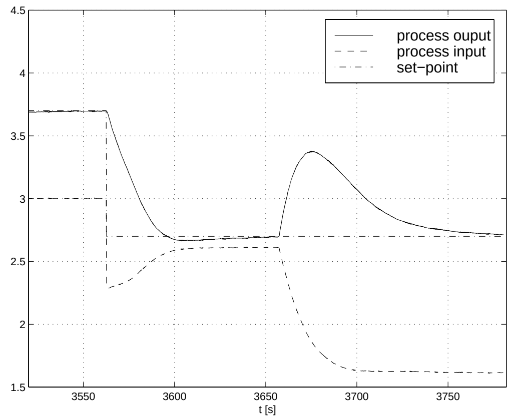

Fig. 10. The closed-loop response to the step change in reference signal and disturbance signal; the controller applied is tuned using the MO method (20).

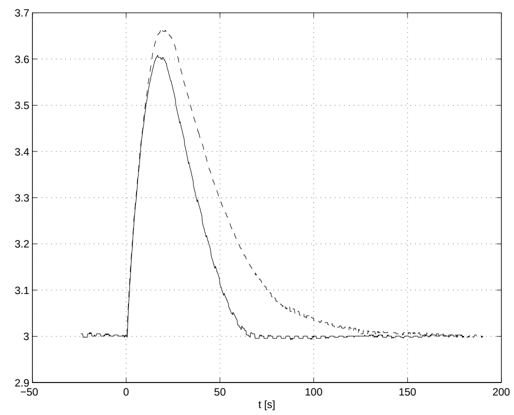

Fig. 11. The closed-loop response of the laboratory set-up to disturbance signal (d=1 at t=0); __ PI controller (21), -- PI controller (20).

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