Windupless PI and PID controllers Tuning

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Abstract: The paper investigates the basic properties of two possible dynamical structures of integrating controllers. They are characteristic with one possible saturated period of control in their responses to a set point step (structures denoted by index „1“), or just with an exponential (in the limit with a step) transient to a new steady state control signal value, without attacking given saturation limits (structures denoted by index „0“). By discussing several possible tuning procedures, the paper is focused on steps required for a reliable controller tuning.

Keywords: PID Control, Disturbance Rejection, Controller Tuning, Magnitude Optimum, Disturbance Reconstruction, Optimal Control.

1. Introduction

It is well known that in controlling systems with a dominant first order dynamics, for an admissible class of input signals the control signal saturation does not cause any serious problems with system stability although it slows down the transients. The basic problems in controlling these systems are mostly caused by the parallel structure of linear PI and PID controllers. These problems are usually increased by the control signal saturation. The resulted performance degradation is denoted as the windup phenomenon. Due to mentioned deficiencies, majority of linear PI and PID controller structures have to use additional anti-windup network in order to reduce undesired effects. Such an approach, however, requires additional tuning of several free parameters. In this paper some aspects of practical implementation and tuning of a new type of fully windupless integrating two-degrees-of-freedom controllers [1] will be considered. These controllers can be tuned independently on reference following and on disturbance attenuation.

2. Basic Controller Structures

In [1] it was shown that a fully windupless PI_1 controller structure for systems with dominant first order plant dynamics can be based on a 2-channel structure consisting of: **(a)** P-controller extended by feed-forward control ensuring desired set point tracking and **(b)** Observer based compensation of input disturbances which guarantees an independent disturbance response and a zero steady-state error. This approach helps to achieve fast transient responses without overshoot typical for the linear PI-structures and does not need any additional anti-windup loops.

In generalising the mentioned approach, a higher-order controller R can be introduced (e.g. PD) instead of the P controller (see Fig.1).

For disturbance compensation it is necessary to estimate the actual plant input, e.g. by using filtered (lead-lag) estimation of the inverse process transfer function. An equivalent input disturbance is then evaluated as a difference of the estimated filtered plant input and of the filtered controller output. The identified disturbance v_f has then to be subtracted from the controller output and the resulting signal to be limited according to saturation limits.

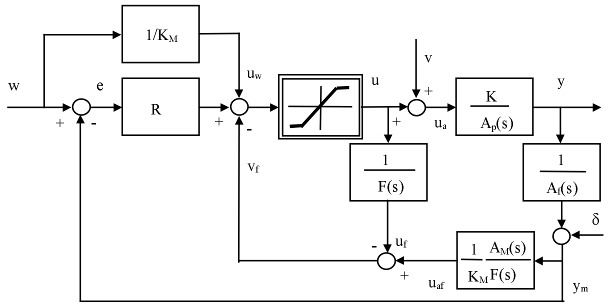

Fig.1. Basic controller R, static feed-forward control and an observer based disturbance compensation for dominant first order plant

For a dominant feedback dynamics, another 2-degrees-of-freedom controller is proposed (see Fig.2). It is composed of a static inverse gain in the feed-forward path and of the channel for reconstruction and compensation of disturbance (Fig.2)

Next, both these structures will be explained with an accent given to the experimental tuning of the controllers for control loops with a gradually increasing degree of complexity.

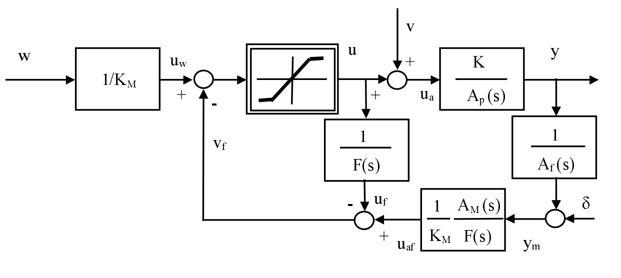

Fig.2. Static feed-forward control and reconstruction based compensation of input disturbance for loops with dominant feedback dynamics

3. First Order Open Loop Approximation

Two cases are possible:

a) The first order plant in the feed-forward path, when it is natural to use the control structure given in Fig.1 with

$$A_f(s) = 1; A_p(s) = 1 + Ts; A(s) = A_f(s)A_p(s) = 1 + Ts$$
$$F(s) = 1 + T_f s$$

This situation (see [1]) corresponds to a windupless PI_1-controller.

b) A dominant feedback dynamics, when the basic control scheme is given by Fig.2 with

$$A_f(s) = 1 + Ts; A_p(s) = 1; A(s) = A_f(s)A_p(s) = 1 + Ts;$$
$$F(s) = 1 + T_f s$$

This basic 2-channel controller is denoted as PI_0 controller.

3.1. Admissible Input Signals

An **admissible control signal** is usually defined by

$$U_1 \le u \le U_2 \qquad (1)$$

In order to maintain the output signal y in a neighbourhood of a chosen reference value w=const, it must be possible to control the polarity of the output signal changes by the admissible control signal. So, for $A_p(s)$ satisfying $A_p(0)=1$ the set of *admissible reference signals* can be restricted by

$$\begin{aligned} &K(U_1 + v) < w < K(U_2 + v), K > 0 \, or \\ &K(U_2 + v) < w < K(U_1 + v), K < 0 \end{aligned} \qquad (2)$$

In controlling systems with unstable $A_p(s)$ (T<0) it is yet necessary to introduce a notion of the *admissible initial state* y_0 [1]. In such a state, the sign of the output derivative has to be changeable by the admissible control.

3.2. P-controller

Setting $T_f \to \infty$, the control loop (Fig.1) behaves like P-controller with gain K_R. The output signal is given as

$$Y(s) = F_w(s)w(s) + F_v(s)v(s);$$
$$F_w(s) = \frac{(1+1/K_{0M})K_0}{1+K_0+Ts}$$
$$F_v(s) = \frac{K}{1+K_0+Ts} \tag{3}$$
$$K_0 = K_R K$$
$$K_{0M} = K_R K_M$$

Obviously, the zero steady state error can only be achieved for $v=0$ and $K_M=K$. Since for $T>0$ and $K_0>0$ the characteristic polynomial $C(s)= 1+K_0+Ts$ is always stable, this plant approximation gives no information relevant for an optimal tuning of K_R.

For unstable plant with $T<0$, the range of possible controller gains would be limited from one side by the condition $K_0<-1$.

However, the controller gain can be limited according to allowed measurement noise amplification.

3.3. Windupless PI$_1$-controller

By placing the loop around the saturation, according to Fig.3, the controller gains integral action with the time constant $T_I=T_f$, where

$$\frac{1}{1-\dfrac{1}{1+T_f s}} = \frac{1+T_f s}{T_f s} = 1 + \frac{1}{T_f s} \tag{4}$$

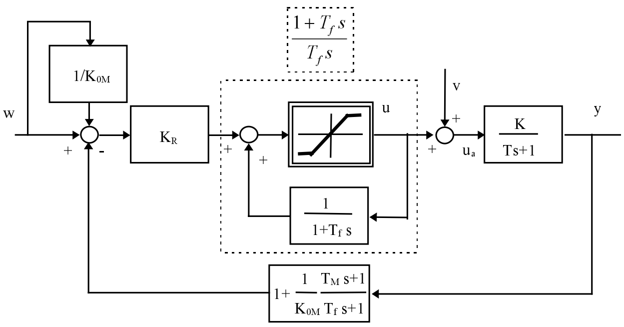

Fig.3. Rearranged structure of windupless PI$_1$-controller; $K_{0M}=K_M K_R$, $K_0=KK_R$.

The idea of anti-windup PI-controller based on a loop around saturation can also be met by other authors [5, 6].

The closed loop is characterised by the following transfer function:

$$F_w(s) = \frac{(1+1/K_{0M})K_0(1+T_f s)}{F_{den}(s)}$$
$$F_v(s) = \frac{KT_f s}{F_{den}(s)} \tag{5}$$

where

$$F_{den}(s) = (1+1/K_{0M})K_0 + [(1+K_0)T_f + T_M K_0 / K_{0M}]s + T_f T s^2$$

Since $F_w(0)=1$ and $F_v(0)=0$, this controller gives a zero steady state error also in the case of a constant input disturbance $v\neq 0$ and of a model mismatch, when $K_M\neq K$ $(K_M\neq 0)$, $T_M\neq T$.

For $T>0$ (stable plants) and $T_f>0$, the closed loop remain stable for any $K_0>0$ and $K_{0M}>0$. In the case of unstable plants with $T<0$ the closed loop stability in the nominal case ($T_M=T$, $K_M=K$) is guaranteed (all coefficient of the characteristic polynomial have the same sign) for $K_0<-1$. It means that the controller gain must not be increased under the critical value ($K_R\leq K_{Rmin}$):

$$K_{R\min} = -1/K \tag{6}$$

A well-balanced dynamics (with equally fast dynamical modes) can be obtained by achieving a double real pole [3] of the closed loop characteristic polynomial. In the nominal case

$$s_{1,2} = \frac{-(1+K_0)T_f - T \pm \sqrt{Det}}{2T_f T} \tag{7}$$

where

$$Det = [(1+K_0)T_f + T]^2 - 4T_f T(1+K_0)$$

From the demand Det=0, it follows

$$T_f = T/(1+K_0) \tag{8}$$

It might seem at the first glance that in the case when $v=0$, and $v_f=0$, the disturbance reconstruction (compensation) channel remains inactive and the loop behaves as the first order one. However, the higher order modes may be initiated by a possible mismatch of initial conditions (e.g. due to the measurement noise) and/or by the model-plant mismatch.

3.4. PI$_0$-controller

In the case of the first order loop dynamics placed in the feedback, Fig.2 can be modified into Fig.4 involving linear PI controller with the proportional gain $K_R=T_M/K_M T_f$ and with the integral time constant

$T_I=T_M$. Together with the input filter it forms a structure denoted here as a PI_0-controller:

$$F_w(s)=\frac{K(1+T_f s)(1+Ts)}{K(1+T_M s)+K_M T_f s(1+Ts)}$$
$$F_v(s)=\frac{KK_M T_f s(1+Ts)}{K(1+T_M s)+K_M T_f s(1+Ts)} \qquad (9)$$

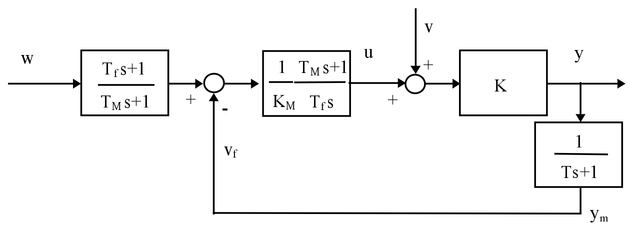

Fig.4. Rearranged structure with linear PI_0-controller and input filter

It is important to note that in the nominal case, due to $F_w(s)=1$, the closed loop behaves like a memory-less system. To eliminate step changes of the control signal (and also to decrease sensitivity to a possible mismatch of K_M and K), the value of T_f in the input filter is being set to zero. Then, the effect of the input filter is equivalent to industrially produced PI controllers with error acting on I-only. From the stability point of view, the filter time constant T_f in controller can take any positive value. However, this value can be limited according to allowed high-frequency gain

$$K_N=K_R=T_M/K_M T_f. \qquad (10)$$

3.5. Summary

The first order open loop estimations are appropriate for design of the basic controller structure. However, they give no information about parasitic time lags involved in each control loop which are of the crucial importance in determining the optimal controller values. So, to make the tuning procedure reliable, one has to use higher order loop approximations.

4. Second Order Open Loop Approximation

In this sub-section, three different situations will be considered:

a) $A_p(s)=1+T_1s$; $A_f(s)=1+T_2s$;
$A(s)=A_p(s)A_f(s)=(1+T_1s)(1+T_2s)$

b) $A_p(s)=1$; $A_f(s)=(1+T_1s)(1+T_2s)$;
$A(s)=A_p(s)A_f(s)=(1+T_1s)(1+T_2s)$

c) $A_p(s)=(1+T_1s)(1+T_2s)$; $A_f(s)=1$;
$A(s)=A_p(s)A_f(s)=(1+T_1s)(1+T_2s)$

4.1. Separate tuning of the P-controller

Disconnecting the disturbance compensation channel in the nominal case (a), for $R=K_R$ from Fig.1 it follows

$$F_w(s)=\frac{(1+K_0)(1+T_2 s)}{(1+T_1 s)(1+T_2 s)+K_0} \qquad (11)$$

For stable time constants ($T_1>0$, $T_2>0$), the closed loop is stable for all $K_0>0$. For $T_1<0$, the closed loop can be stable just for $K_0<-1$ and $0<T_2<-T_1$.

An optimal closed loop gain K_0 can be determined from the requirement of equally fast particular modes of the closed loop dynamics, i.e. by choosing a double real pole of the characteristic polynomial [3] corresponding to

$$K_0=\frac{(T_1-T_2)^2}{4T_1T_2} \qquad (12)$$

The closed-loop step responses are monotonous just for $T_2<T_1$. For $T_2>T_1$ overshoot already occurs - the loop has to be classified as a loop with dominant feedback dynamics and the corresponding control structure (Fig.2) has to be used. The other possibility is the active compensation of the time constant T_2 by the PD-controller.

Although the requirement of a double real closed-loop poles gives sometimes too damped transients, it still does not give the critical value of the controller gain.

4.2. PI_1-controller

In the nominal case (a), for $R=K_R$ and $F(s)=1+T_f s$, from Fig.1 it follows

$$F_w(s) = \frac{(1+K_0)(1+T_f s)(1+T_2 s)}{\begin{pmatrix} 1+K_0+[(1+K_0)T_f+T_1]s+ \\ +T_f(T_1+T_2)s^2+T_1T_2T_f s^3 \end{pmatrix}} \qquad (13)$$

For $T_M=T_1$, $T_1>T_2>0$ (dominant stable plant dynamics), the closed loop remains stable, if
$T_f>0$ AND $T_f>T_1(-T_1+K_0T_2)/[(1+K_0)(T_1+T_2)]$.
For $T_f>T_1T_2/(T_1+T_2)$ the closed loop stability is guaranteed for any K_0, and when $K_0<T_1/T_2$ for any $T_f>0$.

In the case of unstable plants with $T_1<0$ the closed loop stability in the nominal case ($T_M=T_1$, $K_M=K$, $T_f>0$, $T_2>0$) is guaranteed for $K_0<-1$ and $T_2<|T_1|$.

Accepting the condition of the double real pole (12) for the P-controller tuning, the closed loop is stable for any $T_f>0>-4*T_1^2*T_2/(T_1+T_2)^2$ i.e. its value can be determined fully independently and the only restriction is to get an acceptable high frequency noise gain

$$K_N=K_R+T_M/(KT_f). \qquad (14)$$

One idea is that a better dynamics can be achieved by choosing $T_M=T_1+T_2$ what corresponds to the approximation of plant inverse at low frequencies.

This PI_1 controller, which has been derived for the configuration (a), gives acceptable results also in the situation (c), when the magnitude optimum (MO) method [2] yields

$$K_0=\frac{1}{2}\frac{T_1^2-3T_2^2}{(T_1+2T_2)T_2};T_f=\frac{T_1^2(T_1+2T_2)}{(T_1+T_2)^2} \qquad (15)$$

An optimal approach to this situation would already be based on a non-linear PID_2 controller described e.g. in [7].

4.3. Example:

K=2; $T_1=5$; $T_2=1$; distribution c).

1) Linear PI based on MO [4]: $K_{R1}=1.3$, $T_I=5.03s$;
2) Windupless PI_1, Double real pole: $K_{R2}=0.4$; $T_{f2}=2.7778$
3) Windupless PI_1, MO: $K_{R3}=0.7857$; $T_{f3}=4.8611$

While controllers 1 and 3 corresponding to MO yield identical response in linear case (Fig.5a), the linear PI with constrained control signal (Fig.5b) shows typically increased overshoot. But, on the contrary, the overshoot of the corresponding windupless PI_1 C3 has decreased due to the narrower region of linear control. The windupless controller C2, with the proportional gain K_{R2} corresponding to the double real pole (12) with the filter constant T_{f2} guaranteeing the same total high frequency noise gain K_{N2} as in the case of linear PI C1, when it is given directly by K_{R1}, i.e.

$$K_{N2}=K_{R2}+\frac{1}{K}\frac{T_1}{T_{f2}}=K_{R1} \qquad (16)$$

gives obviously the fastest responses both for the reference step, and for the disturbance step. Without the limitation on the noise attenuation, the filter time constant T_2 could be further decreased and the disturbance response could yet be made faster.

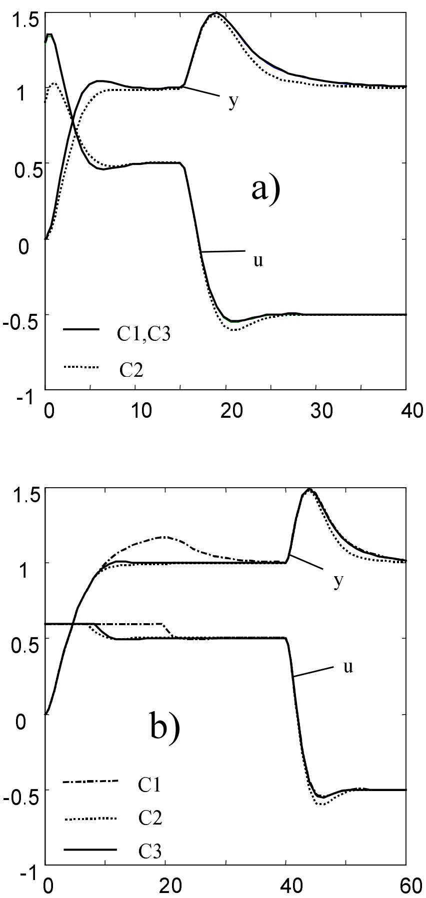

Fig. 5. The closed-loop responses of the second-order process when using three chosen controllers in a) linear case; b) constrained case.

4.4. PI_0-controller

Use of PI_0 controller is recommended in the case of the dominant feedback dynamics, i.e. for configuration (a) with $T_1<T_2$ and for the configuration (b), with $T_1<T_2$. According to Fig.4, for $T_M=T_2$, one gets

$$\text{a) } F_w(s) = \frac{1+T_f s}{1+T_f s+T_1 T_f s^2}\text{, or} \tag{17}$$

$$F_w(s) = \frac{(1+T_f s)(1+T_1 s)}{1+T_f s+T_1 T_f s^2} \tag{18}$$

It means that the loop is stable for any $T_1, T_2, T_f > 0$. Appropriate controller setting could e.g. be based on the demand of equally balanced dynamical modes of the closed loop response, when

$$s_{1,2} = \frac{-T_f \pm \sqrt{T_f^2 - 4T_1 T_f}}{2T_1 T_2};\qquad Det = T_f^2 - 4T_1 T_f = 0 \Rightarrow T_f = 4T_1 \tag{19}$$

With respect to the dynamics of disturbance compensation, this setting will be acceptable just for small values of T_1. Another limitation on the controller tuning could be given by allowed noise amplification specified by (10).

It is to see that this value is smaller as in the case of the PI_1-controller. This may be reason, why in a rough industrial environment the PI_0 controller is used also for the loops with dominant plant dynamics. However, responses achieved are no more the optimal ones. They can be locally (i.e. for specific input, initial condition and saturation limit values) improved by a modified setting and so it might be one of the most serious arguments for existence of the inflation of different optimal settings of linear PI-controllers!

5. Conclusions

The paper investigates the basic properties of two dynamical structures of integrating controllers. Several closed-loop control schemes were proposed. It was shown that the first-order estimations are appropriate for design of the basic controller structure. However, they give no information about parasitic time lags involved in each control loop which are of the crucial importance in determining the optimal controller values. Moreover, it was shown that the design of the P-controller gain is not fully independent on the disturbance reconstruction and compensation channel. Elimination of this dependence requires use of higher-order plant estimations in disturbance reconstruction.

Several experiments by using the second-order process estimation have been conducted. It was shown that the proposed windupless controller structure (PI_1), by choosing appropriate double real closed-loop pole (12), gives the fastest response both for the reference change and disturbance step-change.

References

[1] Huba, M., P. Kulha, P. Bistak and Z. Skachova (1997). Two Dynamical Classes of PI-Controllers for the first Order Loops. *Preprints 2nd IFAC Workshop New Trends in Design of Control Systems*, Smolenice, 293-298.

[2] Vrančić, D., Y. Peng, S. Strmčnik, R. Hanus (1996). A new tuning method for PI controllers based on a process step response. *Pre-prints of the CESA'96 IMACS Multiconference, Symposium on Control, Optimization and Supervision*, Lille, **2**, 790-794.

[3] Oldenbourg, R.C. and H. Sartorius (1944, 1951). Dynamik selbsttätiger Regelungen. 2.Auflage, R.Oldenbourg-Verlag, München.

[4] Vrančić, D. (1998). A New PID Controller Auto-Tuning Method Based on Multiple Integration. *Preprints Int. Summer School RT-Control,* Prague, 126-138.

[5] Kothare, M.V., Campo, P.J., Morari, M. and C.V.Nett (1994). A Unified Framework for the Study of Anti-windup Designs. Automatica, Vol 30, 1869-1883.

[6] Åström, K.J. and T. Hägglund (1988). Automatic Tuning of PID Controllers. Instrument Society of America, Research Triangle Park.

[7] Huba, M. and P. Bisták: Dynamic Classes in the PID Control. Accepted for ACC'99, San Diego, CA.