Practical Guidelines for Tuning PID Controllers by Using MOMI Method

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Abstract – **The magnitude optimum multiple integration (MOMI) tuning method for the PID controllers provides non-oscillatory closed-loop response for a large class of process models. However, one must account for certain additional obstacles that have to be overcome to enable application of the method in practice. A few practical guidelines for performing multiple integrations (MI) from the process step response in practice, for re-tuning controller parameters, and for calculating the parameters of the two-degrees-of-freedom controller in order to improve disturbance rejection performance, are given.**

I. INTRODUCTION

The Ziegler-Nichols tuning rules (Ziegler and Nichols, 1942) were the very first tuning rules for PID controllers, and it is surprising that they are still widely used today. Their popularity lies in their simplicity and efficiency. This is why so many different tuning rules which are based on the same tuning procedures have subsequently been developed (Gorez, 1997).

Following the work of Ziegler and Nichols, a variety of PID tuning methods have been developed. In general, these methods can be divided into two main groups: the direct and the indirect tuning methods (Åström et al., 1993; Gorez, 1997).

The direct tuning methods do not require a process model, while the indirect methods calculate controller parameters from an identified model of the process.

Recently, a new indirect tuning method which is based on an implicit process model was developed (Vrančić et al., 1996). The multiple integrations (MI) method (Rake, 1987; Strejc, 1960) is used for the implicit process identification. However, the areas, calculated by using the multiple integrations from the open-loop process response, are directly used for the calculation of the controller parameters rather than for the process identification in order to meet the so-called magnitude optimum (MO) criterion (Åström and Hägglund, 1995; Hanus, 1975; Kessler, 1955). It was found out that in this way, by using the so-called magnitude optimum multiple integration (MOMI) method, the magnitude optimum criterion can be met for a very large set of process models (low-order, high-order, highly non-minimum phase and/or processes with larger time delays) merely by measuring the process open-loop step response without the need for additional “fine” tuning. The excellent tuning results were also achieved on several laboratory set-ups (Vrančić, 1997; Vrančić et al., 1997; Vrančić et al., 1998b).

However, one must account for certain additional obstacles that have to be overcome to enable application of the method in practice or to improve disturbance rejection for some certain processes. Such problems and corresponding solutions are closely studied in this paper.

II. THE MOMI PID CONTROLLER TUNING METHOD

A magnitude optimum multiple integration (MOMI) tuning method is based on a magnitude optimum (MO) frequency criterion which makes the frequency response from set-point to plant output as close to one as possible for low frequencies.

If GCL(s) is the closed-loop transfer function from the set-point (w) to the process output (y), the controller is determined in such a way that

$$G_{CL}(0)=1 \qquad \left.\frac{\partial^n |G_{CL}(j\omega)|}{\partial\omega^n}\right|_{\omega=0}=0 \tag{1}$$

for as many n as possible (Åström and Hägglund, 1995).

Such criterion results in a fast and non-oscillatory closed-loop time response for a large class of process models.

In order for the MO method to be applied by using the following PID controller transfer function:

$$G_C(s)=\frac{U(s)}{E(s)}=K\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+sT_f}\right), \tag{2}$$

where u is the controller output and e is the controller error (e=w-y), and the following process transfer function:

$$G_P(s)=\frac{Y(s)}{U(s)}=K_{PR}\frac{1+b_1s+b_2s^2+\cdots+b_ms^m}{1+a_1s+a_2s^2+\cdots+a_ns^n}e^{-sT_{del}}\ , \tag{3}$$

an explicit identification of the parameters K_{PR}, a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5, and T_{del} is required (Vrančić, 1997; Vrančić et al., 1997, Vrančić et al., 1998b). However, it is well known that accurately estimating such a number of parameters from real measurements could prove to be very problematic.

However, this problem can be avoided by using the concept of multiple integrations (Rake, 1987; Strejc, 1960). Following Rake, (1987), the following areas can be expressed by integrating the process open-loop step response ($y(t)$), after applying the step-change ΔU at the process input at t=0:

$$A_1=y_1(\infty) \quad \vdots \quad A_k=y_k(\infty)\ , \tag{4}$$

where

$$y_0(t)=\frac{y(t)-y(0)}{\Delta U}$$
$$y_1(t)=\int_0^t[y_0(\infty)-y_0(\tau)]d\tau\ . \tag{5}$$
$$\vdots$$
$$y_k(t)=\int_0^t[A_{k-1}-y_{k-1}(\tau)]d\tau$$

In order to meet quite demanding MO frequency criterion (1), the PID controller parameters can be calculated in the following way (Vrančić, 1997; Vrančić et al., 1997, Vrančić et al., 1998b):

$$T_d = \frac{A_3 A_4 - A_2 A_5}{A_3^2 - A_1 A_5} \tag{6}$$

$$K = \frac{A_3}{2\left(A_1 A_2 - A_3 K_{PR} - T_d A_1^2\right)} \tag{7}$$

$$T_i = \frac{A_3}{A_2 - T_d A_1} \tag{8}$$

Note that the PI controller parameters can be expressed from (7) and (8) simply by applying T_d=0.

Also note that Equations (6) to (8) hold when the filter time constant is fixed to T_f=0. However, choosing e.g. T_f=T_d/10 still does not seriously affect the result of the calculation of the PID controller parameters (see Vrančić, 1997, and Vrančić at al, 1998a, Vrančić et al., 1998b).

III. GUIDELINES FOR PRACTICAL WORK

The previous section showed that the implementation of the magnitude optimum multiple integrations (MOMI) method is very simple and straightforward. Only the process step response has to be measured, and some integrations (summations) to be performed in order to calculate areas A_1 to A_5 (A_1 to A_3 for PI controller). However, there are always some additional obstacles that have to be overcome in order to be able application of the method in practice. In this section, a few practical guidelines for deriving areas from the process step response will be given, as well as some modifications of the tuning procedure where the calculated controller gain is too high, or even negative, or where using a two-degrees-of-freedom controller.

A. Performing multiple integrations in practice

Areas A_1 to A_5 can be calculated from the final values (t=∞) of signals y_1(t) to y_5(t) (4). Of course, in practice it is sufficient to wait until process step response settles. Fig. 1 shows a typical process step response. At t=t_1, a step-change is applied to the process input. The process practically reaches the steady-state value at t=t_{int}, so all the integrations in (5) can be made in the time interval t=[t_1, t_{int}].

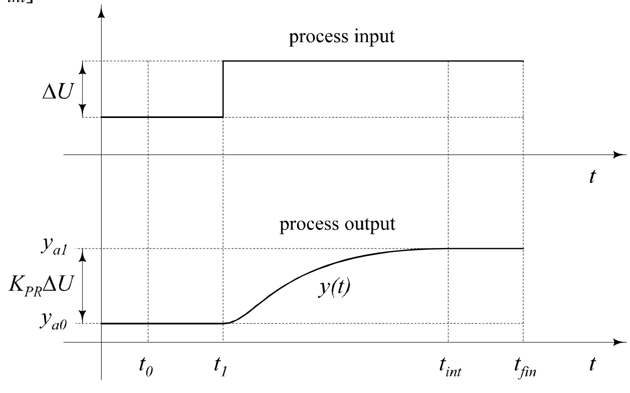

Fig. 1. Process input and output during step-change experiment.

However, relatively small errors in calculating the process steady-state gain (K_{PR}) could lead to relatively large errors in the calculated areas. Such errors are especially noticeable when dealing with a process corrupted by noise. To improve the accuracy of the calculated K_{PR}, the process step response should be averaged in time intervals t=[t_0, t_1] (before making step change) and t=[t_{int}, t_{fin}] (after the new steady-state was already been achieved) in the following way (see Fig. 1):

$$\begin{aligned} y_{a0} &= \overline{y(t)};\ t = [t_0, t_1] \\ y_{a1} &= \overline{y(t)};\ t = [t_{\text{int}}, t_{fin}] \end{aligned}. \tag{9}$$

The process steady-state gain is then simply calculated as:

$$K_{PR} = \frac{y_{a1} - y_{a0}}{\Delta U}\ . \tag{10}$$

Note that y(0) in (5) should be replaced by y_{a0}.

How are the time instants t_0 and t_{fin} chosen? Numerous experiments on several process models and laboratory plants showed that good practical results are usually obtained when choosing:

$$\begin{aligned} t_1 - t_0 &= 0.1 \ldots 0.3 \cdot (t_{\text{int}} - t_1) \\ t_{fin} - t_{\text{int}} &= 0.1 \ldots 0.3 \cdot (t_{\text{int}} - t_1) \end{aligned}. \tag{11}$$

The proposed integration procedure will now be illustrate using an example.

The following process model was chosen:

$$G_P(s) = \frac{1}{(1+4s)^3}\ . \tag{12}$$

A random noise, generated by the MATLAB function RANDN, and amplified by factor 0.05, was added to the process step response. The process output and input signals are shown in Fig. 2. The following time intervals were chosen: t_0=0s, t_1=10s, t_{int}=50s, and t_{fin}=60s. Values y_{a0} and y_{a1} were calculated by averaging process output signal during intervals t=[t_0, t_1] and t=[t_{int}, t_{fin}] (9) which resulted in y_{a0}=-6.97·10^{-4}, and y_{a1}=0.996. Using (10), the calculated process gain was K_{PR}=0.997. Functions y_1(t) to y_5(t) were calculated from (5), where integrations were performed in the time interval t=[t_1, t_{int}]. Areas A_1 to A_5 were calculated from $y_1(t_{int})$ to $y_5(t_{int})$. The following values of the areas and controller parameters were obtained:

$$\begin{aligned} &process: K_{PR} = 0.997,\ A_1 = 11.87,\ A_2 = 93.47, \\ &A_3 = 604.1,\ A_4 = 3433,\ A_5 = 1.762 \cdot 10^4 \\ &PI: K = 0.595,\ T_\text{i} = 6.46 \\ &PID: K = 2.50,\ T_\text{i} = 9.92,\ T_\text{d} = 2.74 \end{aligned}. \tag{13}$$

The ideal values, obtained on the process without noise present, were the following:

$$\begin{aligned} &process: K_{PR} = 1,\ A_1 = 12,\ A_2 = 96, \\ &A_3 = 640,\ A_4 = 3840,\ A_5 = 2.15 \cdot 10^4 \\ &PI: K = 0.625,\ T_\text{i} = 6.67 \\ &PID: K = 2.31,\ T_\text{i} = 9.87,\ T_\text{d} = 2.59 \end{aligned}\ . \tag{14}$$

It is clear that the obtained controller parameters (13) are close to the ideal ones (14).

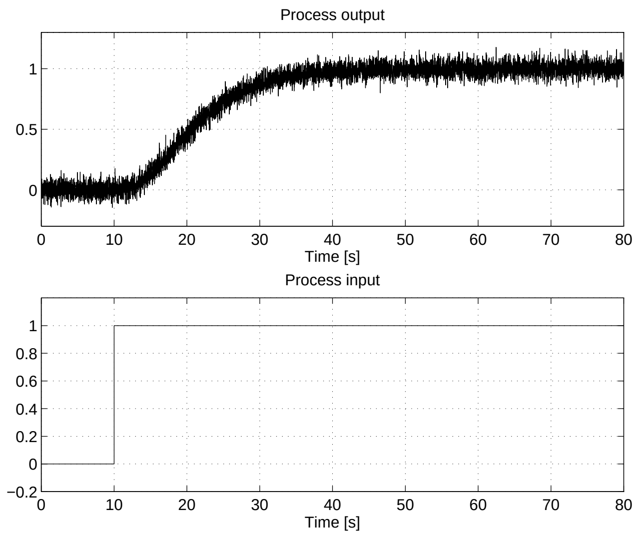

Fig. 2. Process output (y) and controller output (u) during the open-loop experiment on the process with present noise.

B. Re-tuning the controller parameters

In some cases, the controller parameters, obtained by using the MOMI method, have to be re-tuned due to some practical reasons, namely that when tuning the PID controllers for a first-order or second-order process the controller gain is, in accordance with the MO tuning criterion, theoretically infinite. In practice (when there is process noise), the calculated controller gain can have a very high positive or negative value. In this case, the controller gain should be limited to some acceptable value, which depends on the controller and the process limitations.

The remaining two controller parameters can now be calculated according to the limited (fixed) controller gain from (7) and (8):

$$T_i = \frac{A_1}{K_{PR} + \frac{1}{2K}} \tag{15}$$

and

$$T_d = \frac{A_3}{A_1^2}\left[\frac{A_1 A_2}{A_3} - \frac{1}{2K} - K_{PR}\right] \tag{16}$$

if

$$K > \frac{1}{\frac{2A_1A_2}{A_3} - 2K_{PR}} \tag{17}$$

and

$$T_d = 0 \tag{18}$$

if

$$K \leq \frac{1}{\frac{2A_1A_2}{A_3} - 2K_{PR}} \ . \tag{19}$$

When limiting the controller gain of the PI controller, of course, only (15) is used. Note that the proposed re-tuning of controller parameters can also be used in cases when slower and more robust controller should be designed (by decreasing gain *K*), or if a faster, but more oscillatory, response is required (by increasing gain *K*).

The proposed modified tuning procedure will now be illustrated.

The following process model was chosen:

$$G_P(s) = \frac{2}{(1+5s)(1+s)} \ . \tag{20}$$

The multiple integrations were performed on the process step response (*y*), and the following values of the process steady-state gain and areas were obtained from (4) and (5):

$$\begin{aligned} &K_{PR} = 2,\ A_1 = 12,\ A_2 = 62,\ A_3 = 312, \\ &A_4 = 1562,\ A_5 = 7812 \end{aligned} \ . \tag{21}$$

In the next step, the PI and PID controller parameters were calculated from (6) to (8):

$$\begin{aligned} &PI:\ K = 1.3,\ T_i = 5.03s \\ &PID:\ K = \infty,\ T_i = 6s,\ T_d = 0.833s \end{aligned} \ . \tag{22}$$

By fixing the controller gain at *K*=10, and by applying (15) and (16), the following modified PID controller parameters were obtained:

$$K = 10,\ T_i = 5.85s,\ T_d = 0.725s \ . \tag{23}$$

Fig. 3 shows the closed-loop process responses when using the original PI controller and the modified PID controller parameters. It is clear that the closed-loop process response when using such modified PID controller is very good.

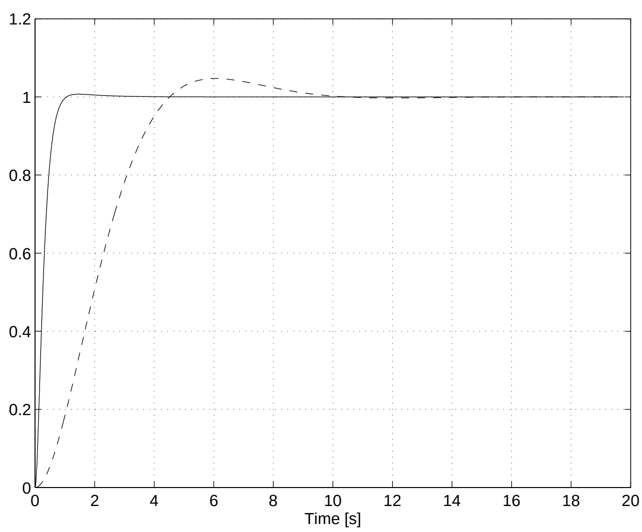

Fig. 3. Process output (y) and controller output (u) during the closed-loop experiment with: __ modified PID controller, -- PI controller.

C. Modified tuning procedure for 2-degrees-of-freedom PI controllers

It is frequently claimed that a drawback of the MO tuning approach is that the process poles are cancelled by the controller zeros. This may lead to poor attenuation of load disturbances if the cancelled poles are excited by disturbances, and if they are slow compared to the dominant closed-loop poles (Åström and Hägglund, 1995).

Poorer disturbance rejection can be observed when controlling low-order processes. In such cases, disturbance rejection can be significantly improved by using a two-degrees-of-freedom PI (PID) controller. However, the controller parameters have to be recalculated according to the changed controller structure.

The controller parameters will be calculated for the simple two-degrees-of-freedom PI controller, shown in Fig. 4 (see e.g. Åström and Hägglund, 1995).

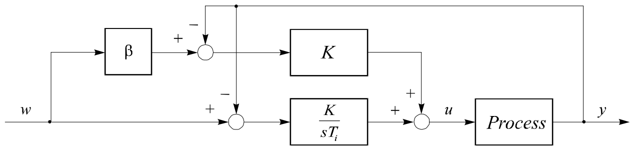

Fig. 4. A two-degrees-of-freedom PI controller.

By following the same tuning objective as given in (1), the following PI controller parameters are derived as a function of parameter β: (see Vrančić (1997)):

$$\begin{aligned} &K^2\left(1-\beta^2\right)\left(K_{PR}{}^2 A_3 + A_1{}^3 - 2K_{PR}A_1A_2\right) + \\ &+ 2K\left(K_{PR}A_3 - A_1A_2\right) + A_3 = 0 \end{aligned} \tag{24}$$

$$T_i = \frac{A_1}{K_{PR} + \dfrac{1}{2K} + \dfrac{KK_{PR}{}^2}{2}\left(1-\beta^2\right)} \tag{25}$$

From (24), the controller proportional gain K can be expressed in the following way:

$$K = \frac{\left(A_1A_2 - K_{PR}A_3\right) - \sqrt{S_1}}{\left(1-\beta^2\right)\left(K_{PR}{}^2 A_3 + A_1{}^3 - 2K_{PR}A_1A_2\right)}, \tag{26a}$$

if $K_{PR}A_3 - A_1A_2 < 0$, and

$$K = \frac{\left(A_1A_2 - K_{PR}A_3\right) + \sqrt{S_1}}{\left(1-\beta^2\right)\left(K_{PR}{}^2 A_3 + A_1{}^3 - 2K_{PR}A_1A_2\right)}, \tag{26b}$$

if $K_{PR}A_3 - A_1A_2 > 0$, where

$$\begin{aligned} S_1 &= \left(K_{PR}A_3 - A_1A_2\right)^2 - \\ &- A_3\left(1-\beta\right)^2\left(K_{PR}{}^2 A_3 + A_1{}^3 - 2K_{PR}A_1A_2\right). \end{aligned} \tag{27a}$$

In case when β=1, or $K_{PR}{}^2A_3 + A_1{}^3 - 2K_{PR}A_1A_2 = 0$, the proportional gain is calculated from (7) by applying $T_d=0$. The remaining question is how to choose the new parameter β? By using optimisation of the closed-loop responses on the reference and disturbance step changes, performed on several process models, the following expression was derived (Vrančić, 1997)[1]:

$$\beta = 0.7 + 0.5\left(\frac{A_1A_2}{K_{PR}A_3} - 1\right), \tag{28}$$

The proposed tuning procedure will now be illustrated. The following process model was chosen:

$$G_P(s) = \frac{1}{(1+40s)(1+4s)(1+s)}\ . \tag{29}$$

The following values of the process steady-state gain and areas were obtained: K_{PR}=1, A_1=45, A_2=1821, and A_3=7.29·10^4. Parameters of the classical PI controller (β=1), and the modified PI controller are calculated from (7), (8), (25), (26a), and (28):

$$\begin{aligned} &PI:\ \beta = 1,\ K = 4.04,\ T_i = 40.05s \\ &\text{modified}\ PI:\ \beta = 0.762,\ K = 4.11s,\ T_i = 22.7s \end{aligned} \tag{30}$$

Fig. 5 shows the closed-loop process responses when using the classical PI controller and the modified PI controller parameters. It is clear that disturbance rejection, when using the modified PI controller, is significantly improved.

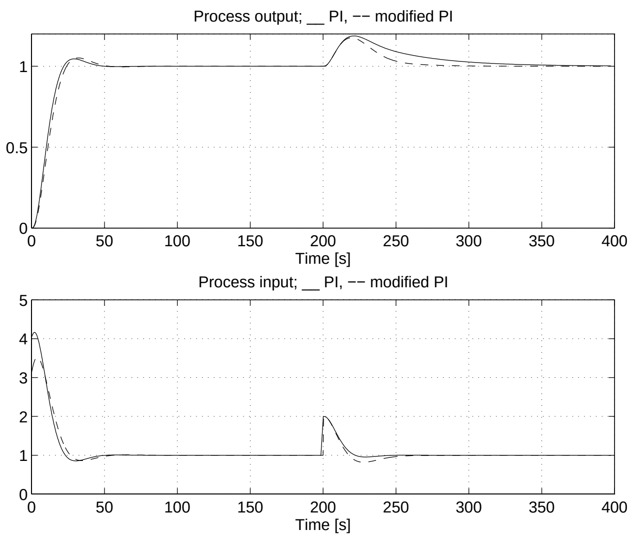

Fig. 5. Process output (above) and controller output (below) during the closed-loop experiment with:
__ classical PI controller, -- modified PI controller.

IV. CONCLUSIONS

The purpose of this paper was to present how to overcome some obstacles in order to be able to apply the MOMI tuning method in practice. Namely, tuning result can become quite sensitive to the process noise, the calculated controller gain can become quite high (positive or negative), and the MOMI tuning method could result in poor disturbance rejection. All of the counted problems were treated in the present paper.

[1] Criteria used for optimisation will be given in the final version of the paper.

It was shown that the tuning result can become relatively insensitive to the process corrupted by noise by properly choosing integration interval of the process step response.

The calculated gain of the PID controller, when applying the MO criterion, could be too high for successful implementation in practice. It was shown that in this particular case the PID controller parameters can be simply re-tuned, according to arbitrary chosen controller gain, without the need for additional process identification stage.

The MO technique may lead to poor attenuation of load disturbances. It was shown that disturbance rejection can be significantly improved by using a two-degrees-of-freedom controller structure.

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