International Symposium on Advanced Model Predictive Control

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Annotation

Over the recent years, a significant progress in the field of Model Predictive Control (MPC) has been achieved. The purpose of the *International Symposium on Advanced Model Predictive Control* is to consider some of the latest achievements in this field. The symposium covers three main topical areas: MPC theory, computational aspects of MPC and MPC applications.

In the paper "Stability of TITO predictive functional control systems", a Two Input-Two Output (TITO) interconnected linear plant with first order plus time delay transfer functions is investigated. The basic algorithm for unconstrained Predictive Functional Control (PFC) is presented. The stability conditions are derived and represented in an explicit form as a multivariable function of the generalized tuning parameters of PFC controller.

The paper "Explicit approximate nonlinear predictive control based on neural network models" suggests an approximate multi-parametric Nonlinear Programming approach to explicit solution of nonlinear MPC problems for constrained nonlinear systems based on neural network models. In particular, the reference tracking problem is considered.

In the paper "Fuzzy model predictive control algorithm, case study", a method for design of nonlinear predictive controller based on a fuzzy model is presented. The Takagi-Sugeno fuzzy model with neural-fuzzy implementation is used and incorporated into the predictive controller. An on-line optimization approach with simplified gradient technique is proposed to calculate the future control actions.

The paper "Stochastic predictive control of a thermoelectric power plant" considers the application of an on-line optimization approach for stochastic nonlinear MPC to the reference tracking control of a combustion plant. The controller brings the air factor on its optimal value with every change of the load factor and thus an optimal operation of the combustion plant is achieved.

The paper "MPC approach in production control" suggests an MPC approach to develop production control systems. Within this approach, different production Key Performance Indicators are identified which are used as referenced controlled variables. Then, an MPC controller is designed to control the production process. The proposed method is applied for the closed-loop control at the production-management level of a polymerization plant.

In the paper "Experience of MPC application in power plants", the experience gained from application of different MPC approaches in a number of Bulgarian thermal power plants, simulation based on real model and some practical applications are presented. The coordinated control of output power and inlet turbine steam pressure are considered as well as the design of the main local control systems – primary superheater temperature, mill-fan pulverization system, combustion process.

The paper "Coordinated model predictive control of a power plant" compares the control performance of the most popular predictive control strategies – Dynamic Matrix Control and Predictive Functional Control. The developed algorithms have been tested on a two input – two output non-linear boiler-turbine model of a power plant at equal conditions. Both of them proved to be among the best control strategies showing good control performance and robustness.

In the paper, "Fuzzy-neural model predictive control algorithm for temperature control of polymer reactor mixture", an algorithm for fuzzy-neural model predictive control is presented. The algorithm is applied for temperature control of the reaction mixture in a polymer reactor. The results with different references show that the proposed control algorithm is applicable for polymerization processes with highly nonlinear characteristics.

The paper "Advanced process control project: A brief overview", considers an Advanced Process Control (APC) technology which aims at process optimization and control performance improvement by implementing MPC algorithms. The features of the APC technology and project implementation are overviewed based on the case studies of APC application at crude distillation units of Russian refineries.

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EXPLICIT APPROXIMATE NONLINEAR PREDICTIVE CONTROL BASED ON NEURAL NETWORK MODELS

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Abstract: Nonlinear Model Predictive Control (NMPC) algorithms are based on various nonlinear models. Among others, an on-line optimization approach for NMPC based on neural network models can be found in the literature. Nevertheless, NMPC with on-line optimization is time consuming. On the other hand, an explicit solution to the NMPC problem would allow efficient on-line computations as well as verifiability of the implementation. This paper suggests an approximate multi-parametric Nonlinear Programming approach to explicit solution of NMPC problems for constrained nonlinear systems based on neural network models. In particular, the reference tracking problem is considered. The approach builds an orthogonal search tree structure of the state space partition and consists in constructing a feasible PWL approximation to the optimal control sequence.

Key words: Nonlinear Model Predictive Control, Explicit Solutions, Neural Network Models

INTRODUCTION

Nonlinear Model Predictive Control (NMPC) involves the solution at each sampling instant of a finite horizon optimal control problem subject to nonlinear system dynamics and state and input constraints [1], [2]. The NMPC algorithms are based on various nonlinear models. Often these models are developed as first principle models, but other approaches, like black-box identification approaches are also popular.

There exist various black-box identification approaches for modeling the nonlinear systems dynamics, e.g. based on neural networks (e.g. [3]), fuzzy models (e.g. [4]), local model networks (e.g. [5]), Gaussian Process models (e.g. [6]). The common feature of the majority of the NMPC approaches based on black-box models is that an on-line optimization needs to be performed in order to compute the optimal control input. Consequently, the computation is time consuming and the real-time NMPC implementation is limited to processes where the time sampling is sufficient to support the computational needs. However, the inefficient computation can be circumvented with an explicit approach to NMPC.

It has recently been shown that the explicit solution to linear constrained MPC problems has an explicit representation as a piecewise linear (PWL) state feedback law defined on a polyhedral partition of the state space [7]. The benefits of an explicit solution, in addition to the efficient on-line computations, include also verifiability of the implementation, which is an essential issue in safety-critical applications. For nonlinear MPC, the prospects of explicit solutions are even higher than for linear MPC, since the benefits of computational efficiency and verifiability are even more important. In [8], [9], [10], approaches for off-line computation of explicit suboptimal PWL predictive controllers for general nonlinear systems with state and input constraints have been developed, based on the multi-parametric Nonlinear Programming (mp-NLP) ideas [11]. It should be noted that the mentioned methods for explicit NMPC are based on *first principle models* of the systems.

This paper suggests an approximate mp-NLP approach to explicit solution of NMPC problems for constrained nonlinear systems described by black-box models. In particular, neural network models as established representatives of the black-box models are considered. Nevertheless, any other black-box model can be used as well. The approach builds an orthogonal search tree structure of the state space partition and consists in constructing a feasible PWL approximation to the optimal control sequence.

The following abbreviation and notation will be used in the paper. The nonlinear model predictive control problem based on neural network model will be referred to as NN-NMPC problem. $A \succ 0$ means that the square matrix A is positive definite. For $x \in \mathbb{R}^n$, the Euclidean norm is $\|x\| = \sqrt{x^T x}$ and the weighted norm is defined for some symmetric matrix $A \succ 0$ as $\|x\|_A = \sqrt{x^T A x}$.

MODELLING OF DYNAMIC SYSTEMS WITH NEURAL NETWORKS

The black-box identification of nonlinear systems is the area which is quite diverse. It covers topics from mathematical approximation theory, estimation theory, non-parametric regression and concepts like neural networks, fuzzy models, wavelets etc. A unified overview of this topic is given in [12]. Suppose the nonlinear system is characterized by a general relationship of the form $y = f(u)$ between an input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^n$. Consider a set of observed inputs $U = [u(1), u(2), \ldots, u(M)]$ and outputs $Y = [y(1), y(2), \ldots, y(M)]$ from the dynamical system. We are looking for a relationship between past observations $[u(t), y(t)]$ and future outputs $y(t+1)$:

$$y(t+1) = F_{NN}(y(t), u(t)) + v(t) \qquad (1)$$

where the term $v(t)$ accounts for the fact that the next output $y(t+1)$ will not be an exact function of the past data. The function F_{NN} is usually parameterized with a finite-dimensional vector of parameters. It is a concatenation of two mappings: one that takes the increasing number of past observations $[u(t), y(t)]$ and maps them into a finite dimensional vector of fixed dimension and one that takes this vector to the space of the outputs. The mentioned finite dimensional vector of fixed dimension is the so called regression vector and its components are called regressors. The regressor vector can be composed in various ways (see [12]). One of the possible choices is that the past values of the observed inputs $u(t), u(t-1), \ldots, u(t-L)$ and outputs $y(t), y(t-1), \ldots, y(t-L)$ are taken as regressors, which defines the so called nonlinear ARX model. The nonlinear mapping from regressor space to the output space can also be of various kinds. In our case we will use mapping with function expansion with basis functions, in particular basis functions of ridge construction which is called a sigmoid neural network. These basis mappings can be convolved with each other and form the so called Multilayer Perceptron (MLP), which is probably the most frequently considered member of the neural network family (e.g. [3]) and can be used as an universal approximator. This particular choice was subjective. Any other choice of regressor vector composition or any other choice of mapping is possible until it enables satisfactory description of the modeled dynamic system. The results given in the continuation of the paper are not limited to MLP approach only.

The class of MLP networks used in this paper is a nonlinear ARX model based on one hidden layer with sigmoid activation function and an output layer with linear activation function. The parameters of this MLP are the weights of the MLP units. After the structure – number of layers and units – is determined, the model parameters are obtained with optimization, based on a chosen cost function. This cost function is most frequently combination of errors between estimated and measured output with input signals. The output prediction can be calculated as:

$$\hat{y}(t+1) = f_{NN}(z(t), \theta) + v(t) \qquad (2)$$

where $z(t) = [y(t), y(t-1), \ldots, y(t-L), u(t), u(t-1), \ldots, u(t-L)]$ is the regressor vector, L is a given lag, θ is a vector containing the weights, f_{NN} is the function realized by the neural network, and $v(t)$ is the prediction error.

FORMULATION OF NN-NMPC PROBLEM AND EXPLICIT APPROXIMATE SOLUTION OF THE RESULTING MP-NLP PROBLEM

Consider the discrete-time nonlinear system characterized by a relationship of the form:

$$y(t+1) = f(z(t)) \qquad (3)$$

where $z(t) = [y(t), y(t-1), \ldots, y(t-L), u(t), u(t-1), \ldots, u(t-L)]$ is the regressor vector, $u \in \mathbb{R}^m$ is the input variable, $y \in \mathbb{R}^n$ is the output variable, and L is a lag. Suppose that we have an output data set $Y = [y(1), y(2), \ldots, y(M)]$ corresponding to an input data set $U = [u(0), u(1), \ldots, u(M-1)]$ and initial condition $y(0)$. Assume that the relationship (3) is approximated with MLP neural network with ARX structure:

$$y(t+1) = f_{NN}(z(t), \theta) \qquad (4)$$

The weights θ of the neural network are determined based on the given input and output data sets.

Define a state vector:

$$x(t) = \begin{cases} [y(t), y(t-1), \ldots, y(t-L), u(t-1), \ldots, u(t-L)], \text{if } L > 0 \\ [y(t), u(t-1)], \text{ if } L = 0 \end{cases} \qquad (5)$$

Thus, $x(t) \in \mathbb{R}^q$ with $q = (L+1)n + Lm$ if $L > 0$ and $q = n + m$ if $L = 0$. Then, the relationship (4) is represented:

$$y(t+1) = f_{NN}(x(t), u(t), \theta) \qquad (6)$$

Suppose the initial state $x(t) = x_{t|t}$ and the control inputs $u(t+k) = u_{t+k}$, $k = 0, 1, \ldots, N-1$ are given. Then, from (6) we can obtain the predicted states $x_{t+k+1|t}$, $k = 0, 1, \ldots, N-1$ which correspond to the given initial state $x_{t|t}$ and control inputs u_{t+k}, $k = 0, 1, \ldots, N-1$:

$$y_{t+k+1|t} = f_{NN}(x_{t+k|t}, u_{t+k}, \theta), \; k = 0, 1, \ldots, N-1 \qquad (7)$$

$$\begin{aligned} & x_{t+k+1|t} = [y^T_{t+k+1|t}, y^T_{t+k|t}, \ldots, y^T_{t+k+1-L|t}, u^T_{t+k|t}, \ldots, u^T_{t+k+1-L|t}]^T \\ & k = 0, 1, \ldots, N-1 \end{aligned} \qquad (8)$$

Here, we consider a reference tracking problem where the goal is to have the output vector $y(t)$ track the reference signal $r(t) \in \mathbb{R}^n$. In the problem formulation, the type of the cost function is like the one used in [7]. Suppose that a full measurement of the state $x(t)$ is available at the current time $t \geq L$. For the current $x(t)$, the reference tracking NN-NMPC solves the following optimization problem:

Problem P1:

$$V^*(x(t), r(t)) = \min_U J(U, x(t), r(t)) \qquad (9)$$

subject to $x_{t|t} = x(t)$ and:

$$y_{\min} \leq y_{t+k|t} \leq y_{\max}, \; k = 1, \ldots, N \qquad (10)$$

$$u_{\min} \leq u_{t+k} \leq u_{\max}, \; k = 0, 1, \ldots, N-1 \qquad (11)$$

$$\Delta u_{\min} \leq \Delta u_{t+k} \leq \Delta u_{\max}, \; k = 0, 1, \ldots, N-1 \qquad (12)$$

$$\left\| y_{t+N|t} - r(t) \right\| \leq \delta \qquad (13)$$

$$\Delta u_{t+k} = u_{t+k} - u_{t+k-1}, \; k = 0, 1, \ldots, N-1 \qquad (14)$$

$$y_{t+k+1|t} = f_{NN}(x_{t+k|t}, u_{t+k}, \theta), \; k = 0, 1, \ldots, N-1 \qquad (15)$$

$$\begin{aligned} & x_{t+k+1|t} = [y^T_{t+k+1|t}, y^T_{t+k|t}, \ldots, y^T_{t+k+1-L|t}, u^T_{t+k|t}, \ldots, u^T_{t+k+1-L|t}]^T \\ & k = 0, 1, \ldots, N-1 \end{aligned} \qquad (16)$$

with $U = [u_t, u_{t+1}, \ldots, u_{t+N-1}]$ and the cost function given by:

$$J(U,x(t),r(t)) = \sum_{k=0}^{N-1}\left[\left\|y_{t+k|t} - r(t)\right\|_Q^2 + \left\|\Delta u_{t+k}\right\|_R^2\right] + \left\|y_{t+N|t} - r(t)\right\|_P^2 \tag{17}$$

Here, N is a finite horizon and $P,Q,R \succ 0$. Note that the term $\Delta u_{t+k} = u_{t+k} - u_{t+k-1}$ in the cost function (17) requires that u_{t-1} is known. With the state vector $x(t)$ generated according to expression (5), this is guaranteed for any $L \geq 0$.

We introduce an extended state vector:

$$\tilde{x}(t) = [x(t),\, r(t)] \in \mathbb{R}^{\tilde{n}},\ \tilde{n} = q + n \tag{18}$$

Let $\tilde{x}$ be the value of the extended state at the current time t. Then, the optimization problem P1 can be formulated in a compact form as follows:

Problem P2:

$$V^*(\tilde{x}) = \min_U J(U,\tilde{x}) \tag{19}$$

subject to:

$$G(U,\tilde{x}) \leq 0 \tag{20}$$

The NN-NMPC problem defines an mp-NLP, since it is NLP in U parameterized by $\tilde{x}$. An optimal solution to this problem is denoted $U^* = [u_t^*, u_{t+1}^*, \ldots, u_{t+N-1}^*]$ and the control input is chosen according to the receding horizon policy $u(t) = u_t^*$. Define the set of *N*-step feasible initial states as follows:

$$X_f = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} \,|\, G(U,\tilde{x}) \leq 0 \text{ for some } U \in \mathbb{R}^{Nm}\} \tag{21}$$

If δ in (13) is chosen such that the problem P1 is feasible, then X_f is a non-empty set.

In parametric programming problems one seeks the solution $U^*(\tilde{x})$ as an explicit function of the parameters $\tilde{x}$ in some set $X \subseteq X_f \subseteq \mathbb{R}^{\tilde{n}}$ [11]. The explicit solution allows us to replace the computationally expensive real-time optimization with a simple function evaluation.

In general, the exact solution of problem P2 can not be found. In [10], a computational method for constructing an explicit PWL approximate solution of NMPC problems has been suggested. The aim of the method is to approximate the optimal solution $U^*(\tilde{x})$ to problem P2 in a hyper-rectangle $X \subset \mathbb{R}^{\tilde{n}}$. It is required that the state space partition is orthogonal and can be represented as a $k-d$ tree. The main idea of the approximate mp-NLP approach [10] is to construct a feasible piecewise linear (PWL) approximation $\hat{U}(\tilde{x})$ to $U^*(\tilde{x})$ on X, where the constituent affine functions are defined on hyper-rectangles covering X. In case of convexity, it suffices to compute the solution of problem P2 at the $2^{\tilde{n}}$ vertices of a considered hyper-rectangle X_0 by solving up to $2^{\tilde{n}}$ NLPs. In case of non-convexity, it would be necessary to include some interior points in addition to the set of vertices of X_0. Based on the solutions at all points, a feasible local linear approximation $\hat{U}_0(\tilde{x}) = K_0\tilde{x} + g_0$ to the optimal solution $U^*(\tilde{x})$, valid in the whole hyper-rectangle X_0, is determined. The details of the approximate mp-NLP algorithm for explicit NMPC can be found in [10].

EXAMPLE

The nonlinear system. Consider the system described by the following discrete nonlinear model with the sampling time $T_s = 0.5$:

$$y(t+1) = y(t) - T_s \tanh(y(t) + u(t)^3) \tag{22}$$

Neural network model identification. The control signal u was generated by a random number generator with uniform distribution. The control signal rate of change was $T_u = 6T_s$, i.e. it is kept constant for 6 time instants or its integer multiples. The number of the input signal samples used for the identification was $M = 200$. Based on the generated data set, the discrete-time system (22) is approximated with a neural network MLP model of ARX structure:

$$y(t+1) = f_{NN}(x(t), u(t), \theta) \tag{23}$$

where $x(t) = [y(t), u(t-1)] \in \mathbb{R}^2$ (cf. equation (5)). The neural network has 5 units with sigmoid activation function in the hidden layer and linear activation function in the output layer. The optimization method applied for identification of the MLP model was the Levenberg-Marquardt method [3]. A validation control input signal was generated by a random number generator with uniform distribution and rate of change (T_u=8T_s) that is different from the one used for the identification signal. Results on the validation signal showed that the obtained neural network model sufficiently accurately models the system.

Design of explicit reference tracking NN-NMPC controller. The mp-NLP approach described in the previous section is applied to design an explicit reference tracking NN-NMPC controller for the system (22) based on the obtained MLP model (23). The following control input and rate constraints are imposed on the system:

$$-1 \leq u \leq 1\ ,\ \ -0.5 \leq \Delta u \leq 0.5 \tag{24}$$

The prediction horizon is $N = 8$ and the terminal constraint is:

$$\left\|y_{t+N|t} - r(t)\right\|^2 \leq \delta \tag{25}$$

where $\delta = 0.001$. The weighting matrices in the cost function (17) are $Q = 10$, $R = 1$, $P = 10$. The NN-NMPC minimizes the cost function (17) subject to the MLP model (23) and the constraints (24)–(25). According to (18) the extended state vector is $\tilde{x}(t) = [y(t), u(t-1), r(t)] \in \mathbb{R}^3$, which leads to a 3-dimensional state space to be partitioned. The latter is defined by $X = [-1.2, 1.2] \times [-1, 1] \times [-0.7, 0.7]$.

The partition has 661 regions and 18 levels of search. Totally, 24 arithmetic operations are needed in real-time to compute the control input (18 comparisons, 3 multiplications and 3 additions).

The performance of the closed-loop system was simulated for the following set point change:

$$r(t) = -0.7, t \in [0;50], r(t) = -0.2, t \in [51;100]$$
$$r(t) = 0.2, t \in [101;150], r(t) = 0.7, t \in [151;200] \tag{26}$$

The resulting closed-loop response is depicted in Fig. 1 and Fig. 2.

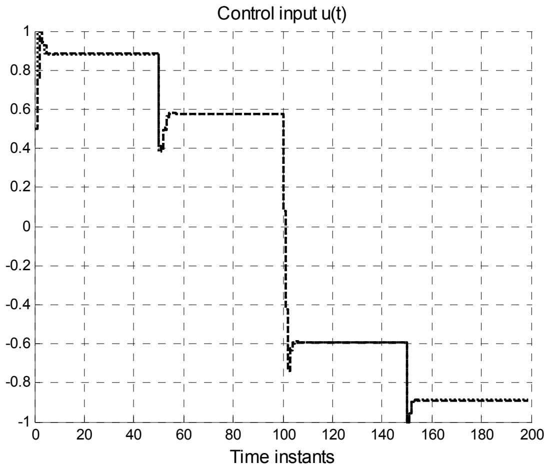

Fig. 1. The control input (the dashed curve is with the approximate explicit NN-NMPC and the dotted curve is with the exact NN-NMPC).

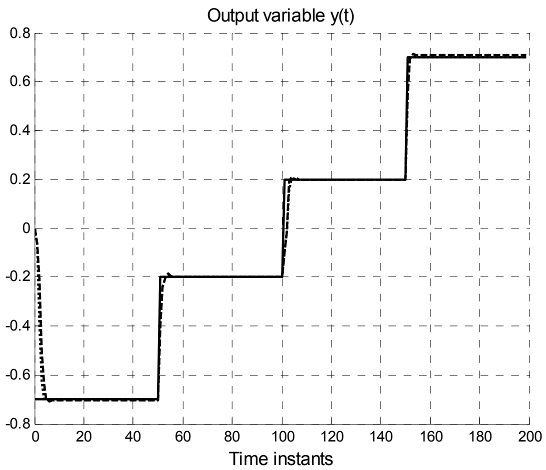

Fig. 2. The output variable (the dashed curve is with the approximate explicit NN-NMPC, the dotted curve is with the exact NN-NMPC and the solid curve is the set point).

The results show that the exact and the approximate solutions are almost indistinguishable.

CONCLUSIONS

In this paper, an approximate mp-NLP approach to explicit solution of reference tracking NMPC problems based on neural network models is developed. The future work would include the extension of the approach to nonlinear systems described by other types of black-box models.

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